JEREMY

ISSUE 1, 2024

PHYSOC

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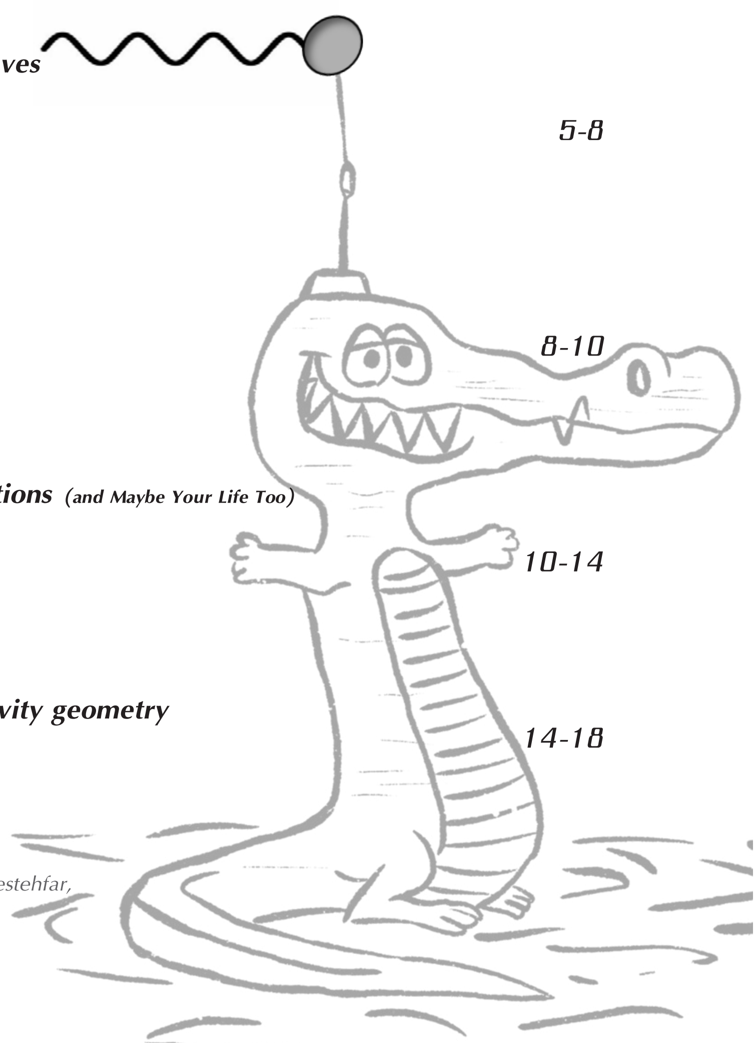


Jeremy Team: Peter Lavilles, Angela Liang, Mani Shayestehfar, Imasha Fernando, Jasmyn Curry, Murray Jones, Aruna Sathyanarayanan Balaprabhavathi

PHYSOC

JEREMY

Issue 1, 2024 *The Physics Society Magazine*

Welcome to Jeremy!

"Oh, Jerry, don't let's ask for the moon. We have the stars."

As semester begins and the air dings with the buzz of friends long-awaited, as people are met and lecture halls fill with the sound of learning and excitement, have a look around yourself. The lake in Victoria Park ripples and swirls, the Clock-tower bells chime from metal on metal, the Quadrangle spires point up to stars shining through clouds and city lights, and if in the midst of it all you find yourself asking '*why does this happen?*', then you are doing physics. Physics is for everyone, for we have all glimpsed from time to time a childlike curiosity of the wondrous world we live in. Whether you are a seasoned physicist or someone who hyperventilates more than an asthmatic hamster from the first whiff of a maths equation, *Jeremy* is here for you. As you thumb through this magazine you are an explorer, an adventurer of new landscapes that anyone can walk, and who knows what discoveries await?

100 Years of the Physics Building

By MIKAELA CHEN

These are some of the stories these corridors, laboratories, stairways, and lecture halls have to tell.

Physics building during construction 1923-1925 and Physics building today. Photograph provided by Tim Bedding.

In 1923, work had just begun on the new Physics building, which the Sydney Morning Herald described as "what will be the finest School of Physics in Australia, and, perhaps, in the Southern Hemisphere".[1] Two years later, in March 1925, the then small Department of Physics officially moved into the newly built Physics building, sharing it with the Department of Mathematics and the Cancer Research Project. The outside of the building commemorates the year 1924, which is why we are celebrating the 100th anniversary this year.

So now a (quasi) century since the beginning of the Physics building, *Jeremy* looks back at its history with the help of Professor Robert (Bob) Hewitt – affiliated with the School of Physics since his student days in the 1960s and who has held many staff roles since, and some thorough scouring of the University of Sydney online archives.

Back to the beginning

Symmetrical, squat, and cream-washed, the Physics building was designed by the University's first architecture professor Leslie Wilkinson, the mastermind behind the University's masterplan. The Physics building constituted part of this plan, designed to have "axes and open attractive views from many points", according to Wilkinson.[2] Before Wilkinson's new design, the tiny physics department had been teaching in part of what is now Badham building.

[1] Sydney Morning Herald, 5 April 1924, pp18. "*Physics School. New building. Finest in Southern Hemisphere*".
[2] University of Sydney Physics Building brochure for Open Day 1984.

Air raid trenches being dug in front of the Physics building. 1940s. REF-00050156

The first alteration to the new Physics building came with WWII, which left almost nothing unchanged in Australia. The basement was enlisted to build and refurbish optical equipment for the military. The rest of the building trained officers for radar stations set up for the war. Meanwhile, the hockey field on the Physics building's front steps was carved out with air raid trenches.

Big projects and their legacies

After WWII, "the University of Sydney was desperate to rejuvenate physics because it was the flavour of the month with the bomb and all the developments during the Second World War," said Robert Hewitt.

Rejuvenation came in the form of Professor Harry Messel. Just 30 years old and with a glowing reference from Erwin Schrödinger, Messel was appointed Head of Physics in 1952. It was the beginning of his influential and passionate 35-year tenure.[3] Very soon eleven new academic staff were appointed to add to the existing four, and the Physic building itself received some renovations it direly needed.

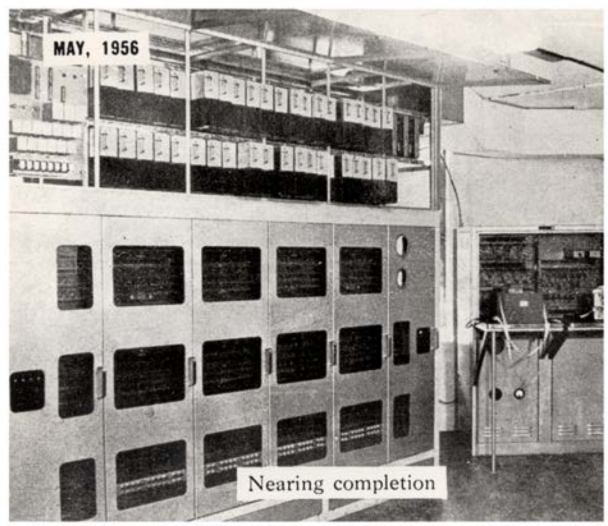
In 1954, Messel procured a donation from Sir Adolph Basser to build SILLIAC, the Sydney version of the Illinois Automatic Computer. SILLIAC was born from ILLIAC, the University of Illinois's computer and the fastest in the world outside of the military at the time. It was the second computer built in Australia and a powerful, exciting one for its time.

"The idea was that Sydney would build [SILLIAC] and if they made any improvements to the design, they would pass them on back to ILLIAC and vice versa," Robert said.

"It was a very important machine. It trained all of the people who were significant in Australian computing. It did all the engineering calculations for the Snowy Mountains scheme. It worked out where to put all the telephone exchanges in Australia."

SILLIAC was significant for its time, even if its capabilities don't hold up today.

"SILLIAC had a memory of one kilobyte. It had 48 little cathode ray tubes which were about 30 centimetres long with a diameter of about 10 centimetres. Each of those cathode ray tubes had 1024 dots which could be on or off. The 48 tubes were the 48 bits of the word. So it had 1024 48-bit words. And it had about 4000 valves. But it was quite a reliable machine."



Front view of SILLIAC. REF-00014271

Around the same time, Messel also established the Nuclear Research Foundation (now the Physics Foundation), the first of its kind in Australia. The Foundation had many uses, but one of them was creating the International Science School (ISS), a program that brings together talented high school students from across the world for two weeks every two years. The students receive lectures from world-renowned scientists and tour research facilities with all expenses covered.

The very first ISS for senior high school students was in 1962 (it was trialled with high school teachers for a few years before

[3] https://www.sydney.edu.au/science/schools/school-of-physics/harry-messel.html

that), with guest lectures given by Hermann Bondi, Ronald Bracewell, and the controversial Wernher von Braun. 108 boys and 45 girls attended, all of them Australian except for one student from New Zealand.[4] The International Science School continues strong today and the next one is due in 2025. It is now truly international, with students coming from Britain, Canada, China, India, Japan, Malaysia, Thailand and the USA. The gender divide has also evened out to roughly equal male and female students most years.

Wernher von Braun giving a lecture at ISS 1962. Photograph provided by Tim Bedding.

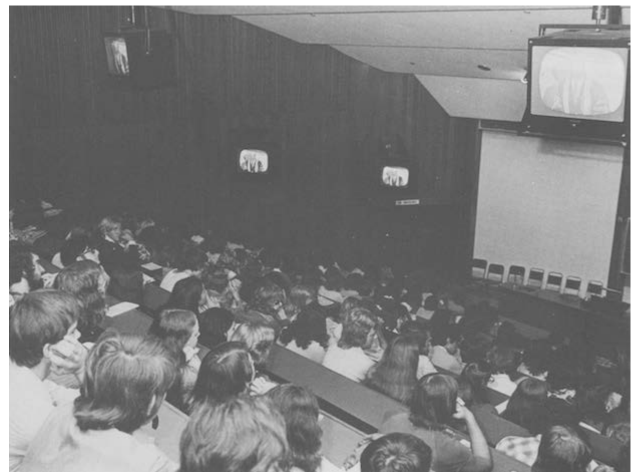
Being a Physics student in the 1960s

In the meantime, the student population studying physics continued to rise. The increase had started post-WWII, when returning servicemen were offered free university education, and continued in the following decades. Robert remembers his own time as a physics student at USyd in the 60s.

"I came in 1961 as a first year student. My first lecture was in the lecture theatre that no longer exists, one down on the ground floor where the big glass windows are. That was physics lecture theatre three, and Harry Messel gave me my lecture in there."

"It was quite a spectacle because he had on the front bench a metal tray with a glass of water, and on the board were all of the pictures for the lecture already drawn in chalk. Then precisely at five past the hour he walked in and they locked the doors and he started talking and gave his lecture. He had one of the senior staff doing the lecture demonstrations for him. And then at precisely five minutes to the hour they unlocked the doors. We never saw anything like that anywhere else in the university."

Soon the science student population had gotten so large it became difficult to fit everyone into the lecture rooms. This was probably why in 1968, the School of Physics began to give first-years recorded lectures through black and white TVs hung from the rafters in Carslaw. Only distinction level students had live lectures in PLT 8 in the Physics Building (the Slade Theatre today). Unsurprisingly and as we who have made it through Covid-induced online learning would understand, these televised lectures were not popular with the students. They didn't last long, though they did cause a temporary decline in Physics enrolments.



Carslaw TV lecture circa late 1960s-early 1970s. Photograph provided by Robert Hewitt.

Not only were the lectures different from today, the labs were too.

"The physics syllabus in those days was very different. All of the equipment you used essentially had to be built by you or gotten second hand from somewhere else. When they closed the tram network, for example, they had a lot of the devices that the driver would use to control the speed of the tram, and they were just variable resistors. So when the trams all closed down the University managed to get those as scrap. When I was a youngster, when we went to buy shoes, you would stand in the little x-ray machine and look down and see whether your foot fitted nicely into the shoe. But these were giving you massive doses of radiation and so they were banned. So the University got all of those. All the equipment was basically stuff that had been scrounged or had been built with cheap components. I didn't see a cathode ray oscilloscope, for example, until I got to third year."

[4] ISS Archives from https://www.sydney.edu.au/science/industry-and-community/community-engagement/international-science-school.html

1960 Physics 1 Laboratory. Photograph provided by Robert Hewitt.

Closing thoughts

In more recent years, the Physics building has gained a shining new neighbour, the Sydney Nanoscience Hub. It boasts more than 25 laboratories and the ability to tightly control those environments with state-of-the-art technologies.[5] Where the Physics building held Australia's best computers of yesterday, scientists in the Nanoscience Hub are researching tomorrow's quantum computers, among many other exciting projects.

So next time you cross the hockey field to get to the Physics Building, maybe you'll feel the ghost of air raid trenches underfoot. Push open the front doors and look up. Framing the edge of the vestibule ceiling are names of individual prominent scientists, most of whom were still living when the building was in construction in 1924 (except for Hertz who died in 1894). All of them have now passed away, and it is a reminder of how many years this building has seen. It's a cliché, but if these walls could talk, they would have so much to say. So in commemoration of the Physics building's 100th birthday, take some time to walk around these corridors, listen to the walls talk, and see what bits of history you can find.

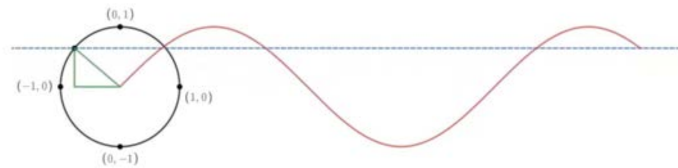
[5] https://www.sydney.edu.au/nano/about/facilities/sydney-nanoscience-hub.html

Sphere Collision Part II: Breaking the Waves

By MURRAY JONES (THE GUY WITH THE HAT)

* Part I can be found on our website usydphysoc.org.au

Someone shouts 'wave!' and the physicists reach for their sinusoids.



It's almost instinct at this point. All through high school, and now throughout our uni degrees, we've all been told that that squiggly line right there, that sinusoid, is more or less the *definition* of a wave.

Dig a bit deeper, and you might unearth some equations to go with it – perhaps the Wave Equation, perhaps Schrodinger's Equation.

$$\frac{\partial^2 u}{\partial t^2} = c\,\frac{\partial^2 u}{\partial x^2} \qquad i\hbar\frac{\partial}{\partial t}|\Psi\rangle = \hat{H}|\Psi\rangle$$

Now, these may look like some very grandiose creations out of the depths of calculus – and indeed they are! These are two of the most powerful equations that physics has to offer. Between them they can model everything from ripples on a pond to radioactive decay, but look a bit closer, and you'll realise they're actually both not much more than a couple of sinusoids in a trench coat.

Now, that's all well and good for ripples on a pond ...

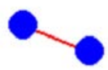
But ponds aren't the full story. The physicist's humble sinusoid is only a *low energy approximation* of a wave. We can do better.

What actually creates a wave?

The usual physics answer would be to say waves are a mathematical construct spat out from the geometry of a circle – but today we're looking for something a little less approximate than that. We could instead say that waves come from the interaction between wind and current at the surface of the ocean – but that would be swinging too far in the other direction. We're still physicists. We like a nice general explanation. So, we need a middle-ground:

> *"Waves are a property of springs."*

If you've ever played with a slinky, this won't seem like too strange a concept. Let's start easy and conjure ourselves a spring:



Nothing fancy here. Just two spheres connected by an ideal spring. How about we make things a bit more exciting and give them some friends:

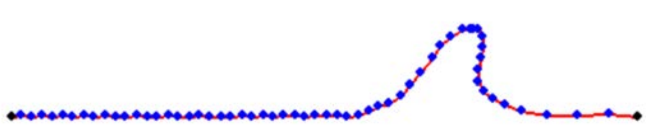
Above: A row of spheres connected by springs. The black spheres are constrained to only move vertically, to prevent the springs from collapsing whole arrangement into a tiny little puddle.

Now let's give one of those spheres a bit of a kick. If we kick it vertically, we can get ourselves a transverse wave, if we kick it horizontally, we can get a longitudinal wave.

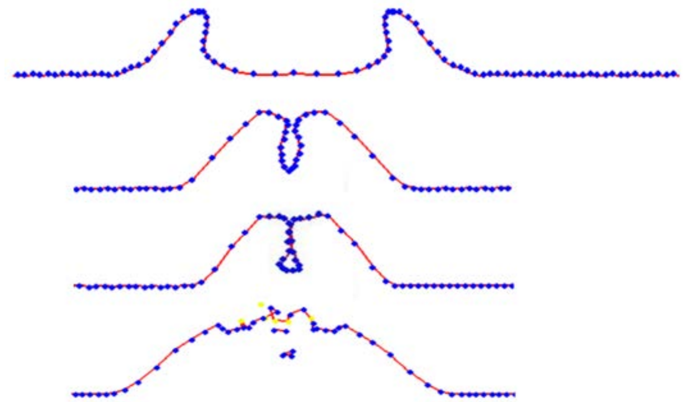
Left: A longitudinal (left and right) wave, such as sound.

Right: A transverse (up and down) wave, such as a ripple in a pond.

Yada yada nothing new there. Nothing we couldn't make by stacking up a whole bunch of sinusoids in a Fourier Series, if we so desired. Let's up the anti and hit our spring with both of those at once: A transverse wave, and a longitudinal wave, on top of each other.



Sorry Mr. Fourier, but that right there is a wave behaviour that no amount of wiggly little sinusoids can model! Now how about we cough up some concave spring collision logic and send two of these waves at each other, from opposite directions:



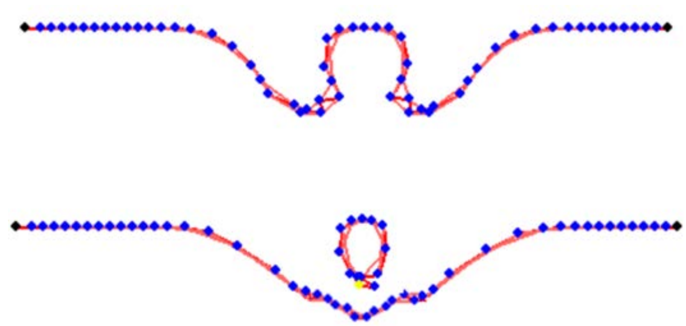
Have you ever dropped a drop of water into a pond, watched the ripples that form, and wondered...

> *'what would happen if I made the ripples first, and sent them inward? Could they combine with each other to splash up a drop?'*

Doesn't matter if you haven't wondered that. You're wondering it now! Point being: The answer is YES, and that's exactly what we just saw happen in our wave on a spring! Now, admittedly, that droplet we just formed wasn't a very exciting one. Currently our springs can only attract spheres, never repel them, so any droplet bigger than three spheres will almost immediately break up into smaller ones. We could fix this by manually introducing a concept of 'pressure', to force enclosed loops of springs to keep their volume constant, but that'd give our system yet another parameter; so instead we'll slightly change the geometry of our setup to deter sharp kinks in our spring:

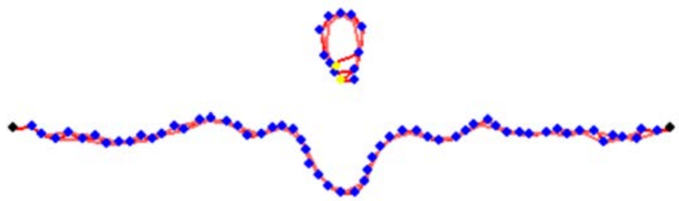
Left: Red springs have nominal length x, orange springs have nominal length $2x$. Thus, the repulsion from the orange springs will cause the system to remain a close to straight as possible.

Right. Let's launch our two waves at each other again, this time our new spring geometry:

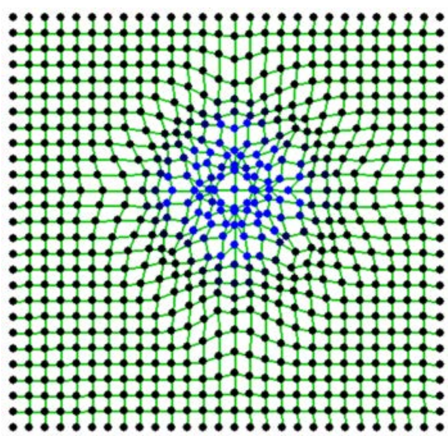


Now we've got two waves that collide to form a nice stable droplet. Yeehaw!

If we add some convex spring collision logic and an acceleration toward the centre line (the line between the two black dots), we can get a droplet that bounces almost indefinitely, resonating on the surface of our spring.



For our original 1D spring, I manually created the longitudinal wave, by setting a horizontal starting velocity to some of the spheres. This made the whole effect bigger and easier to see on the plots, but even if I hadn't done that – even if I'd started with a purely transverse (up and down) wave, there would still have been longitudinal (left and right) movement. This is easier to see if we add a dimension:

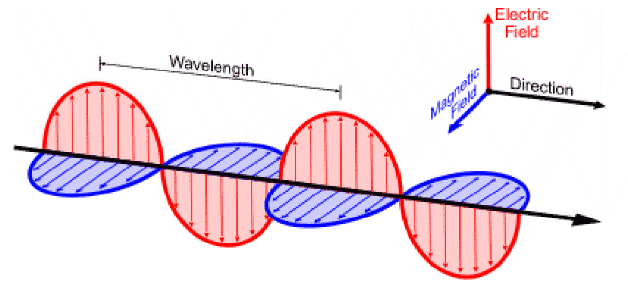


Here, I simply picked a single sphere, and give it a honking massive kick directly upward, right at the start of the simulation. Even though there was no longitudinal motion to begin with, we still see a stretching in the springs before they start moving upward.

- The upward motion stretches the springs.
- The stretched springs retract, pulling spheres closer together.
- The momentum of the spheres compresses the springs as the spheres bunch together.
- This in turn causes upward motion, as the springs expand forcing spheres out of the plane.

So we have a kind of wave with two components. One component of the wave creates the other component, and vice versa, and when two such waves collide, we can form particles.

Does that sound like something familiar?



Correct answer: No. No it doesn't...

Those of you who know a tad about physics are probably grasping in the general direction of Maxwell's theory of Electromagnetic Waves right about now and thinking *'Wait what? You've been working up to this for the whole article!?'*.

Well my friends, the realm of physics is a deep one, and there's a lot to be learnt. Including that Jeremy has plot-twists! For the folk that have been sitting comfortably on a Plank Theory sofa for the last four pages: *Touché*. This waves-on-springs model is lovely. At least, I think it is. However, as my stats lecturer once said:

> *"All models are wrong, but some are useful."*

This model is a way to remove our reliance on the indestructible box we had in *Part I*, and for that it works pretty well. With our sphere bouncing an a lattice of springs, we could model everything from the double slit experiment to the photoelectric effect.

However, as the Plank sofa sitters are itching to point out, we could never match the *Plank-Einstein relation*.

$$E = hf$$

Our model looks beautiful from a distance, but no matter how hard we try, this simulation of a sphere on a lattice of springs will never match the energies of real particles, measured in experiment. For that, we'll have to dive deeper. We'll add dimensions like guacamole to nachos as we delve head-first into the world first unveiled by Louis de Broglie, in *Part III* of ***Sphere Collision***.

What are Numbers?

By PETER LAVILLES

"Out of an infinity of designs a mathematician chooses one pattern for beauty's sake and pulls it down to earth."

- Marston Morse

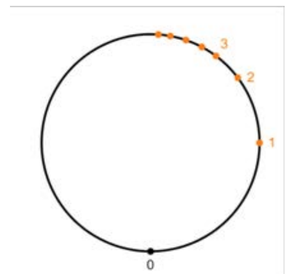
The winds howl, waves crash onto the boat. Against the silhouette of night a figure is seen, teetering dangerously over the boat's edge as members of the Pythagorean cult surround him. Suddenly he is grabbed, lifted high, and thrown overboard. For he was a rebel, someone who divulged a terrible secret that was never meant to be known. He had discovered that irrational numbers exist.

What are numbers, and how do we know they exist? The Pythagoreans were certainly convinced that they are real entities, enough to drown someone over. Numbers are also the foundation of physics, the study of the universe. If we are to say that quarks and electrons exist, then we must say that the numbers which *define* these elementary particles in their truest mathematical form also exist.[6]

Surprisingly, most of the numbers that we have encountered boil down to a very simple object: they are nothing more than structured points on the surface of a sphere.[7] Humans need a way of making this sphere tangible and have constructed a very efficient method for arriving at all of its points; the way individual numbers appear is the outcome of a human codification of this abstract spherical structure. A solid construction is necessary, however, so how do we go about doing it?

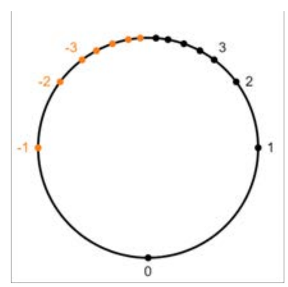
We can't begin from nowhere, so we start with an assumption that cannot be proven (an axiom): *there exists a number* 0*.* Remarkably, 0 is the only number we need to assume exists. Every other point on the sphere can be reached by assuming the existence of operations, ways of reaching new numbers given old ones. 0 is our foundation, our base point from which we can launch into a conquest of the numbers.

The first operation we assume is *counting.* To see what this is let us visualise our intermediate goal, a circle (once this circle is constructed, we can revolve it in space to get our final sphere). The operation of counting says that we can push the number 0 a discrete step away to form a new number, then another step away, and so on without ever stopping. This generates an endless supply of so-called *natural numbers*, which appear arbitrarily cramped the closer we get to the north pole. Counting can be thought of as a kind of warped, non-uniform rotation around the circle, where each natural number hops over to the one closest. The number '2' is code for 'rotate our base point 0 twice around the circle'.



The natural numbers (counting).

The next operation we assume is *negative counting*. This is exactly the same process as before except that we reverse the direction; non-uniform rotations are now clockwise instead of anti-clockwise. This kind of inverse operation can be seen as an expression of symmetry. Placing a vertical mirror inside our circle we see that every natural number has a unique reflection, its very own shadow partner, with the exception of 0. It is so empty that it is its own reflection.



Vertical symmetry (negative counting).

Now we introduce a seemingly innocent operation, which imbues the natural numbers with a profound structure. This is

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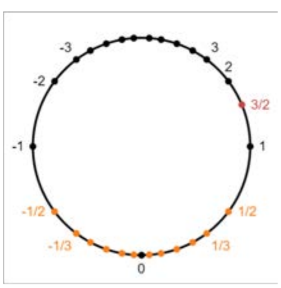
[6] We believe electrons exist since they cause light which we can perceive, even though we can never perceive an electron in and of itself. Numbers exist in the same way in that they cause the electron which causes the light, despite an inability to perceive numbers without the help of symbolic codifications or *constructions*.

[7] This statement comes with two big caveats. Firstly, I am only considering the most basic system of numbers used in physics, the (extended) field of complex numbers. There exists a rich landscape of numbers which are not as fundamental to physics, including the p-adic numbers, quaternions, surreal numbers, etc. Secondly, I am only considering the simplest spherical manifestation of this 'abstract object'. There exist topologically different versions of this object (called Riemann surfaces), like tori.

the operation of *multiplication*, counting with a change in unit. If we write an expression like 3×2 (three lots of two), what we have done is make our base point 0 hop around the circle three times, but each hop is worth double; counting is just a special case of multiplication with a normal unit. This new operation doesn't immediately generate any new numbers, but it tells us something fascinating: every natural number can be broken up into atoms called *prime numbers*. These atoms are unbreakable entities, like $2, 3, 5, 7, 11$. Every other natural number is a molecule that can be reached with a unit change, like 4 which can be broken down into 2×2. We also see that 0 is special in that it sucks in numbers like a black hole; once a number is multiplied with 0 it is stuck there, you can't multiply your way out of 0 again.

So far, we have constructed the natural numbers, we have made them separable into atoms, and we have given them reflective partners. Now we create a new symmetry by placing a horizontal mirror inside our circle, which is achieved through the operation of *division*. This gives every number a new kind of upside-down partner, an inversion of itself. This symmetry is balanced around the number 1, which is so slim that flipping it upside-down leaves it looking identical. Division is the opposite of multiplication; instead of doubling or tripling our unit of counting, we are now splitting up our unit into smaller pieces. Since there is an endless supply of natural numbers, we now have an endless supply of increasingly smaller units. Zeno is quaking in his shoes as his hare's strides become shorter and shorter, apparently unable to catch up with the tortoise.



Horizontal symmetry (division), and the first new fraction $3/2$.

We are now going to perform a feat which leads to an explosive generation of numbers on the circle. We are going to allow multiplication between our original naturals and these new smaller units. For example, what is three lots of half units? It has to be greater than two lots of halves, which is 1 (since 2 and $1/2$ are inversions), and it has to be less than four lots of halves, which is 2 (since 4 breaks up into 2×2). We have made a discovery, a number called $3/2$ lying in the cracks between 1 and 2. Our stockpile of small units lets us repeat this procedure, finding more and more fractions within the cracks until our circle looks entirely covered in points. Just as Lisa Meitner split the atom so we have split our atomic numerals, dissolving their discreteness into numbers of any size we like. The set of numbers we have generated so far are very pleasing, firstly because they appear to completely cover our circle, and secondly because they are extremely ordered. They show predictable patterns when expressed as decimals; for example, the fraction $3/11$ can be written as $0.272727...$. It is here that the Pythagoreans wanted to stop, for they believed these nicely behaving numbers could describe all things in the universe. [8]

Their belief was to crumble under a shocking discovery, however. The apparent order of Nature was an illusion, an ironic flirtation, for even though it looks like we have completely covered the circle there are also uncountably many gaps in our scaffolding. For example, any fraction can be organised into one of two sets: those whose square is less than 2 and those whose square is greater than 2. The point separating these two sets is a hole, for no fraction can square to 2. [9] Another way of saying this is that if there is a number which squares to 2 then it is a chaotic, *irrational* number. Its decimal expansion will never settle down, it can never be predicted.

What we have done so far is create a 'nearly-circle'. As the old saying goes, "if it looks like a duck and quacks like a duck, then it probably is a duck". The most natural thing to do is to fill in these holes, to polish our roughly-hewn structure until it is perfectly smooth. The operation we now assume is the action of completion, where anything we can arbitrarily approach with fractions, including these holes, is a number. Remember that it is the abstract circular object and later the sphere which are what truly exist. The dodgy identity of these irrational numbers as 'holes' is no fault of their own but rather a side-effect of our particular construction which began with counting. In fact, irrational numbers like π and e are essential to Nature, cornerstones in the pillars of the universe. Perhaps an alternative construction of the perfectly continuous circle is possible in which every number has an immediate solidness

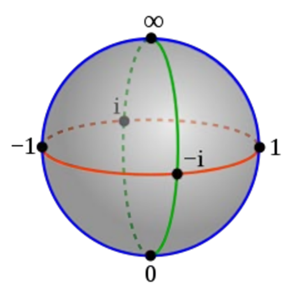
[8] The Pythagoreans came to terms with fractions as *ratios*, or relations between natural numbers.

[9] A *square* is the multiplication of a number with itself. The proof of the statement is by contradiction: assume that $(a/b)^2 = 2$ where a and b have no common atoms (or prime factors). Then $a^2 = 2b^2$, so a is even; let $a = 2p$. Then $2p^2 = b^2$ so b is even. a and b have the common factor of 2, which is a contradiction. Therefore, our assumption was false; $\sqrt{2}$ is inexpressible as a fraction.

to it, or perhaps this apparent awkwardness is unavoidable. Regardless, this is the way humans have come to terms with numbers, and unless a maverick comes up with a construction of the numbers that does not begin with counting, this is likely the way it will stay.

You may have noticed a special kind of gap that we haven't yet mentioned, which has been there from the start: the hole at the north pole. Let's fill it in and call it ∞, 'the point at infinity'. It is an entity that can never be reached but only approached through counting. It is in some sense the inverse of 0 but Nature has decided to make these polar opposites incompatible, for they cannot be multiplied together. ∞ resists being tied down by our construction, flying within the clouds of the abstract but remaining visible.

We have now reached our intermediate goal, a complete circle. All that is left is to revolve this around to get to our final sphere. To do these we need a new operation which pushes numbers into another dimension, and we denote this operation as multiplication by i. We can see that if we start at the number 1 and do two of these 90^o pushes, we end up at -1. In other words, $i \times i = -1$, or i is the square root of -1.



The (Riemann) sphere of numbers.

Finally, we have arrived at the landscape in which numbers live, the surface of a sphere. The job of the scientist is to find how this abstract object and the real world interact, through measurements and experiments. Nature, the ever-mischievous trickster, has decided to draw a veil between the two; an experiment can only measure the height (latitude) of a number on the sphere, but not the direction in which it faces. Despite quarks and electrons having the entire surface as their arena to perform upon, humans can only catch them in the act on part of the stage. The discoveries in quantum physics from the last century have shown that the universe's existence rests upon an object of perfect smoothness and symmetry that can never be completely seen: a sphere of numbers.

How Squiggles Can Revitalize Your Equations (and maybe your life too)

By CALEB CLARK

There is a common notation used in higher level maths/physics that often confuses students, obfuscates the core details of expressions or derivations, and honestly is just a little ugly. This is known as 'index notation', which features expressions like $\hat{e}_i \epsilon_{ijk} A_i B_k$ for something as simple as a cross product, and it gets particularly hairy when vector calculus is concerned. Often teachers may say "here are eight-to-twelve vector calculus identities for you to memorize (or excruciatingly prove)". In my experience this is not long after learning about what derivatives are – certainly you have not had time to actually investigate the identities – and so you are left to consider them as long, arduous identities that just happen to emerge from a behemoth and chaotic mess of mindless misery and tedious calculation.
But does it have to be this way? What if I told you there is a way to reduce $12+$ banal identities into ONE fundamental identity, and that you can get to the rest with just a bit of doodling? Introducing, a pictorial representation of vector calculus and tensor networks – or as I like to call it, squiggles. By the end of this article, you will be convinced that there is more to math than just numbers and symbols – there is kind of an aesthetic creativity to it as well.

Basic Concepts

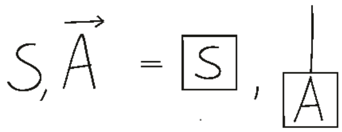
Let us begin by exploring the basic concepts of vector calculus, and a basic drawing which can represent each of these.

Vector, Scalar, Gradient

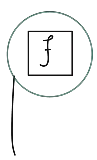
Firstly are the numbers we are familiar with, for example temperature, length, or mass, which are called 'scalars'. We will write these as just a letter inside a box.

Next is something called 'vectors', which we can think of as multiple scalars packaged into one – for example to describe your position or velocity, you need to give one component for each $x-$, $y-$ and $z-$ directions, and the vector is $\vec{r} = \begin{pmatrix} x & y & z \end{pmatrix}$. We will generally notate these as $\vec{A}$ or $\vec{B}$ in normal notation, but for this notation to emphasize the fact that there is one 'free index' – here it is just columns – we will write it with one line sticking out of it. A quantity such as a matrix with two indices of columns and rows (hence with two numbers needed to specify which entry of the matrix we are in) would have two lines.

So a scalar s, then a vector $\vec{A}$ we would write like this:



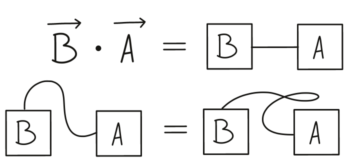
and to multiply them we could just remove the comma and have them float next to each other. Now for these vectors, we need to have a notion of a derivative. For this we use the gradient, which represents the direction and slope of steepest ascent of a scalar function. The gradient, and its interaction with other elements, will be the main consideration of this article. Since the gradient acts on something, we will notate something being differentiated as being inside the bubble. For example, ∇f, where f is a scalar function, we would write as



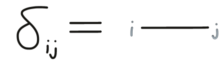
In order to talk about more complicated derivatives such as $\nabla \cdot \vec{A}$ or $\nabla \times \vec{A}$, we will need a way to represent $\cdot$ and $\times$. Let us look into one such way.

Dot Product Machine

Since two vectors are represented as boxes with lines sticking out, the most logical notation for $\vec{B} \cdot \vec{A}$ would be just to 'contract' the lines (using similar language to the index notation). Note that this line and boxes can do whatever they want in terms of movement, as long as the connections remain the same.

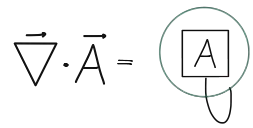


Now we can do something which may seem weird to anyone not used to the index notation, or even to those used to it, but it is one of the most powerful ideas here. We can take any drawing from above, and remove the vectors, leaving only the line between them. This gives us the dot product 'machine', also known as the Kronecker delta δ_{ij}. (Plugging our vectors in, using the index notation, would be $A^i \delta_i^j B_j$.)



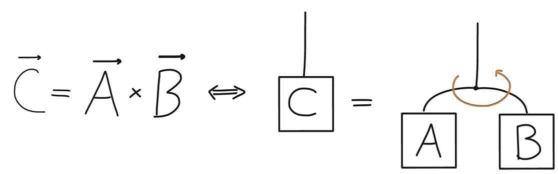
(1)

We can now use this to form divergence of a vector field $\nabla \cdot \vec{A}$ by just connecting the lines of a vector, and the line of the balloon ∇ (which we will begin to colour aqua):



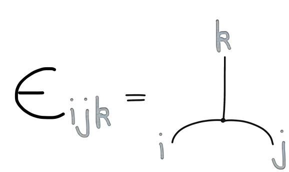
Cross Product Machine

Now that we've got the dot product under our belts, let's see if we can fit the cross product in there as well. The cross product takes in two vectors and outputs a third, so we will need two lines for input, and one for output. This means



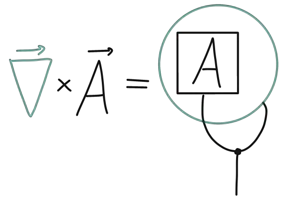
Of course, there must be some asymmetry in the cross product, and we can include this by emphasising that inputs at the terminal (the vertex of the lines) must be read anti-clockwise – emphasised by the orange arrow in the above equation. The discontinuous swapping of two lines in the machine would then introduce a minus sign. We are otherwise still allowed to move boxes or lines around in any continuous deformations, as long as we don't change the order of the connections at the terminal.

We can similarly remove the vectors and just leave the 'machine' exposed; in this case we have the cross product machine, also known as the Levi-Civita symbol, ϵ_{ijk}.



Again now that we have the relevant operation, we can use it to form a derivative, in this case the curl $\nabla \times \vec{A}$. This would be written by linking the $\vec{A}$ line counterclockwise of the ∇

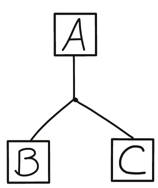
balloon line, giving



Some Symmetries That Are Suddenly Obvious

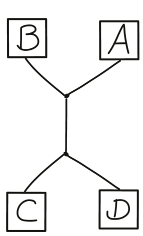
Now, if there is anything a good form of mathematical notation should be able to do, it should be to capture elegant symmetries present in the logical structure of nature and codify them into a logical, intuitively obvious framework. Even without talking about any more concepts, we have already got symmetries slipping out between our fingers. Take a look!

For example, one can take the dot product machine in Equation (1) and rotate it 180^o; this looks the same, which captures that $\delta_i^j = \delta_j^i$, or $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$. We could also take the expression $A\cdot(B\times C)$ (as long as we give it back eventually). Geometrically, any avid 3Blue1Brown viewer knows almost by definition of the cross product that this is going to be the volume of the parallelepiped spanned by A, B, C, and so it should be unchanged upon permutations of those quantities. Unfortunately, the current notation does not make this immediately obvious. However, let us take a look at the graphical notation:



Clearly we can rotate this by $120^o = \tau/3$, and not change the value, so the symmetry is built in to the graphical notation!

Let us try another example. If I take the expression $(A\times B)\cdot(C\times D)$, what symmetries might it have? There seems to be one or two up to permuting the cross products, but nothing else appears immediately obvious, even upon staring. However, once we peek at the graphical notation

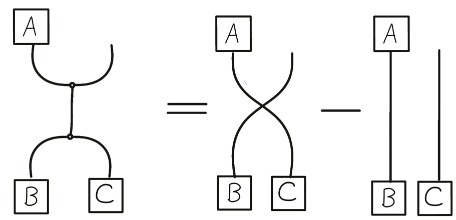


(2)

we see that we could also interpret it as $A\cdot(B\times(C\times D))$, or as $D\cdot((A\times B)\times C)$, or as any number of analogous expressions. This is (only part of!) the magic of this notation: symmetries in expressions are much easier to discover at a simple glance, without sludging through cumbersome vector formulae or overwhelming indices.

The Only Identity You Need to Remember

It turns out that the expression from Equation (2) also shows up in a key identity of vector calculus. The identity as follows (which unfortunately we must show by writing out a few lines of algebra, but the graphical notation is much more memorable than the algebraic forms we will see shortly):

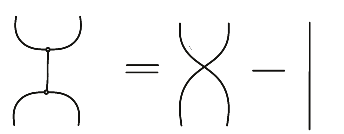


(3)

Writing this algebraically, we have

$$\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C},$$

which already seems more opaque, but not too difficult to memorise; however we could also extract the vectors from Equation (3) and just leave the core of the identity



(4)

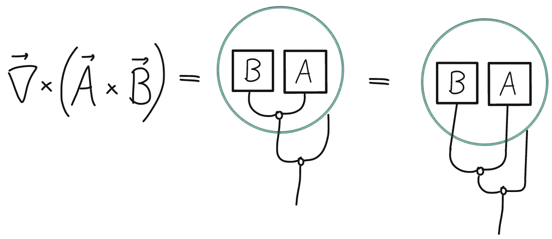
which is algebraically written as

$$\epsilon^{ijk}\epsilon_{klm} = \delta_l^i\delta_m^j - \delta_m^i\delta_l^j$$

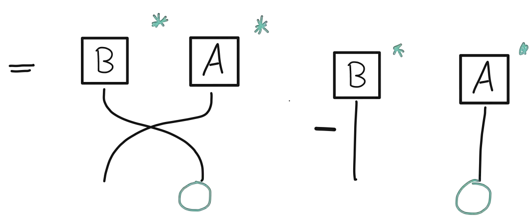
– yuck! Not only this, but we can remove or add only some of the vectors as we needed, to extract various other identities. This identity will be immensely useful for the computation in the next section and many other vector computations one can carry out.

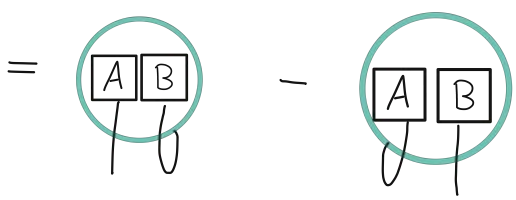
An Example of Practical Squiggle Usage

To give an example of how to practically use this notation, let us step through a computation of $\nabla\times(A\times B)$. First we draw our diagram, noting that we can pull cross (or dot) product machines out of the differentiation balloon since they are constant. Connecting the product $A \times B$ to the balloon line via another cross product machine gives

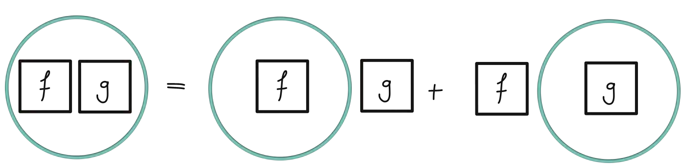


Now for ease of connecting to what we have seen earlier, let's move elements around but preserve the connections (this is allowed, as we said), and write it in a form resembling the LHS of Equation 3. It is a good idea to put a star or dash on the A and B to remember that they are being differentiated, while we momentarily pull them out. After doing this we use Equation 3, and then re-insert the vectors in the balloon.

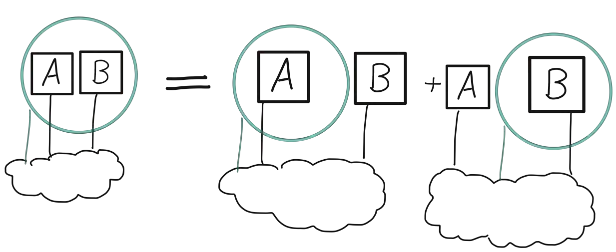




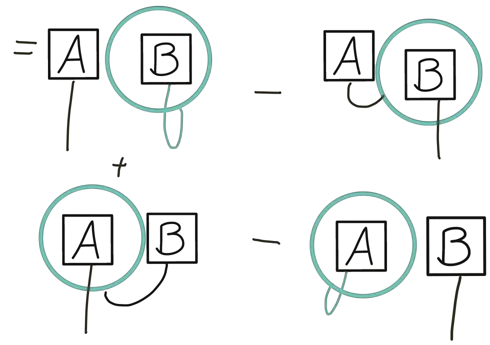
Now we will use an evolved version of the product rule. The basic product rule would look like



When there are vector lines involved, it turns out that we can keep these intact - an illustration of this is below.



Applying this product rule and keeping lines intact to each term we had above, we get



which we can convert back into vector notation by interpreting each term using the previous rules. This gives us the identity as a whole being

$$\nabla \times (\vec{A} \times \vec{B}) = (\nabla \cdot \vec{B})\vec{A} - (\vec{A} \cdot \nabla)(\vec{B}) \\ + (\vec{B} \cdot \nabla)(\vec{A}) - (\nabla \cdot \vec{A})\vec{B}$$

which looks much more intimidating written out compared to how fun it was to derive.

Now what we have seen here are all the tools you need to derive many other identities, such as $\nabla \cdot (A \times B)$ and $\nabla(A \cdot B)$. These and plenty other exercises in [1] could prove fun to attempt if you happen to be stranded on an island with only a stick to draw lines in the sand.

Conclusion

As we have mentioned, so far it may seem like this is merely a prettier or more elegant way to write the same calculations we are used to from before; however there is much more power in the depths of this language. For example, by taking Equation (4) and being more frugal with the vectors we stick on, leaving 2 or 3 lines ('free indices') exposed in each term, we can approach some tensor identities. Additionally the very nature of the language as free-flowing and visual invites newfound creativity and the exploration of new shapes and hypotheses of their possible meanings.

My personal view is that one should be comfortable with reading index notation, but often when completing intermediary calculations on a pen and paper it is far easier (and more fun!) to doodle it out and see what happens. It also comes back and makes reading index notation more enlightening and understandable. Others [2] also believe that the use of these tools in the classroom could make vector calculus and other related classes much more entertaining and exploratory, rather than potentially dry and shrouded behind arduous calculations.

But hey, don't let me keep you any longer; go get squiggling!

Acknowledgements

This article is largely based on [2] which is highly recommended reading, with more detail and examples than this article; the supplementary material [1] delves into much more depth, not only giving many practice problems but also diving into tensors, quantum mechanics, and more. A masterful book featuring similar graphical language is [3].

References

[1] Joon-Hwi Kim, Maverick Oh, and Keun-Young Kim. An invitation to graphical tensor methods: Exercises in graphical vector and tensor calculus and more, 01 2020.

[2] Joon-Hwi Kim, Maverick S. H. Oh, and Keun-Young Kim. Boosting vector calculus with the graphical notation. American Journal of Physics, 89(2):200–209, February 2021.

[3] Roger Penrose. The Road to Reality : A Complete Guide to the Laws of the Universe. Random House, London, 2005.

Investigating Laser properties through Cavity geometry

By RIYA RAIZADA

Abstract

Lasers are oscillators of electromagnetic waves that output monochromatic, collimated beams. A Laser Cavity is a resonator made of mirrors. It is essential for the positive feedback of radiation, enabling stimulated emission in lasers. This experiment investigated how the geometry of a cavity can be altered to optimise properties of the laser such as stability, or how it can be used to determine inherent attributes such as wavelength. Unstable points were found for three different cavity configurations and occurred when the resonator length was larger than the radius of curvature of the mirrors. Wavelengths of the He-Ne laser and Alignment laser were found to be $642 \pm 7\ nm$ and $539 \pm 8\ nm$, matching theoretical values within uncertainties. Sources of error were determined to be human error or misalignment errors. Future work can be done in investigating the regions of stability for other configurations, or investigating other geometric dependencies on laser cavities such as laser output power, beam waist and divergence.

Introduction

Light Amplification by Stimulated Emission of Radiation, or LASER for short, is an oscillator of electromagnetic waves that outputs monochromatic, collimated beams with a Gaussian intensity profile. Its invention is based primarily on the principle of Stimulated Emission. One of the main components that contributes to the operation of lasers is the laser cavity, which enables the light amplification process. The aims of this experiment were to experimentally obtain the stability thresholds for different laser cavity configurations and compare them to theoretical values, and additionally to make use of diffraction and laser cavity geometry to find the wavelength of different lasers.

Main Components of the Laser

First predicted by Einstein in 1917, Stimulated Emission is the emission of photons generated by the decay of electrons from high to low energy states catalysed by interactions with incident photons. Emitted photons have the same direction, phase and frequency as the incident photon. This is different to spontaneous emissions, which can occur at any time and in any direction [7]. Stimulated Emission requires population inversion, which is when more electrons populate higher energy levels than the ground state of an atom [12].

For the function of a laser, the population inversion must be maintained for an extended period of time. This can be artificially achieved by providing the atom with energy through the means of a pump source [14]; the transfer of energy can be electrical, chemical, etc.

While lasing can occur with just two levels, realistically more complex atomic configurations can assure optimum lasing. As such, the four level laser (4LL) is typical for laser construction.

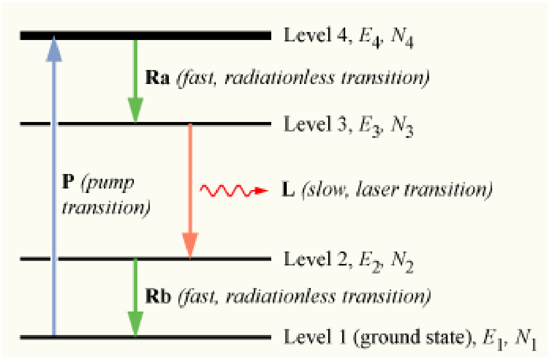


Figure 1: Schematic of Four level Laser. Level 2 and 3 are the lasing levels, and the pump transition occurs from level 1 to level 4. On the timescale of the laser transition, the transitions between L4-L3 and L2-L1 occur almost instantaneously, so the lower lasing level is always empty. Source: [8]

In this configuration the lasing levels are separate from the pump action. The lasing material is called the gain medium and its properties, such as the spacing between levels, can determine the wavelength of emissions.

The Light Amplification process is another critical element of lasers. This is the process of amplifying the amplitude of a wave through positive feedback in an optical resonator. As mentioned before, Stimulated Emission cannot occur without incident photons. Lasers are thus constructed so that the gain medium is placed within a resonator cavity in order to partially reflect photons back toward the medium and collide with the atoms in order to further stimulate emission [11]. Additionally, lasing is dependent on an initial spontaneous decay to kickstart the simultaneous emission.

Typically, Laser cavities consist of a set up of partially reflective mirrors that reflect select wavelengths back toward the lasing medium and cause constructive interference at certain integer multiples of the wavelengths (modes). These wavelengths are amplified, while waves not in phase are lost through destructive interference [13].

Geometric properties of the cavity, in particular the radius of curvature of the mirrors and the distance between them (resonator length), determine key features of the laser such as phase, beam width, stability and divergence.

Laser Stability

The stability of the laser refers to the smooth intensity output of the beam. The ratio between radius of curvature b_i of the mirrors and resonator length d determines the lasing stability. In particular, parameters g are defined as [16]

$$g_i = 1 - \frac{b_i}{d} \tag{5}$$

For each mirror, the stability criterion is given as

$$0 < g_1 \cdot g_2 < 1$$

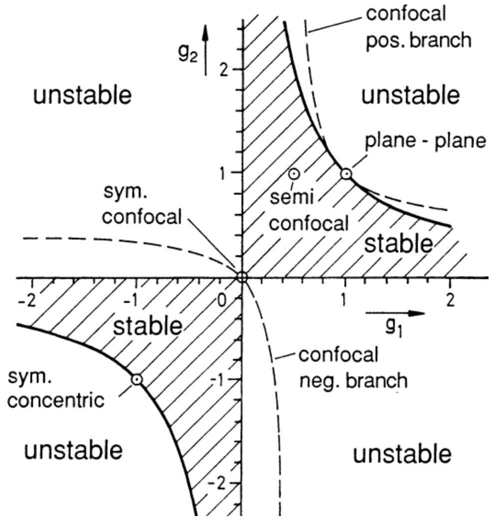


Figure 2: Stability criterion graphed as regions for g_1 v/s g_2 Source: [16]

For any configuration of the laser cavity, instability of the beam occurs when the beam is allowed to escape the resonator [10].

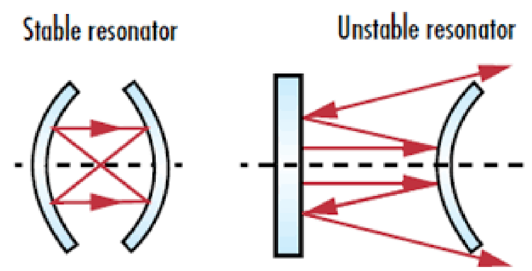


Figure 3: An example of laser beam geometry in a stable and in an unstable resonator. In particular, instability occurs when the laser beam is reflected out of the cavity. Source: [6]

Diffraction Geometry

The geometric make up of the laser can be exploited in order to determine key features of the laser. Diffraction is one way in which geometric analysis of the beam can be conducted. Incidence of a laser beam on a diffraction grating causes constructive and destructive interference. The diffracted intensity distribution appears as the Fourier transform of the beam on a screen situated such that it satisfies the far field approximation - when the distance between the grating and the screen z_{gs} is significantly greater than the grating size l.

Additionally, the optical path difference is dependent on properties of the diffraction grating, and constructive interference occurs when the diffracted beam is in phase (i.e is proportional to the wavelength).

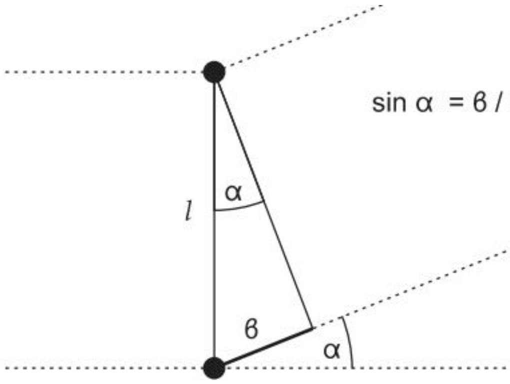


Figure 4: A diagram of the geometry of the path length difference δ generated by a diffraction grating of width l and at an angle of α. Source: [16]

We have that

$$\delta = n \cdot \lambda = l \sin(\alpha) \tag{6}$$

where n is the mode, λ is the wavelength of the laser and α is the angle at which the grating diffracts the incident beam to the first diffracted spot. Trigonometric analysis of the diffraction set up gives a useful relation for α

$$\tan(\alpha) = \frac{y_{s1}}{z_{gs}} \tag{7}$$

where y_{s1} is the distance between the undiffracted spot and the first diffracted spot.

Significance

The laser cavity is one of the most important components of the laser as it has a direct influence on the stability and efficiency of a laser [15]. With lasers having such widespread applications to diverse fields such as spectrometry, medicine, telecommunications, etc., the need for efficient lasers is insurmountable [9]. These needs motivate the investigations into geometry and stability of laser cavities.

Method

Apparatus Set up

Here is a diagram of the experimental apparatus:

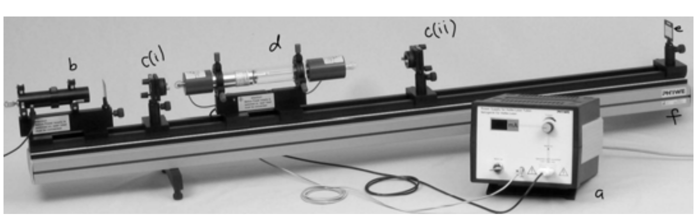


Figure 5: Experimental Setup. Obtained from [16] The components are: (a) power supply, (b) alignment laser, (c)(i) & (ii) cavity mirrors, (d) He-Ne Laser tube, (e) right diaphragm and (f) optical rail.

General alignment procedure

The ignition of the beam required precise alignment of all components, such that the diffraction of the beam within the laser tube was limited to a high degree. The alignment procedure involved shining a green alignment laser with wavelength 532 nm through the cavity and laser tube, which contained the Helium-Neon gain medium, onto a diaphragm. Instances of diffraction presented as concentric rings around the central beam. The inclination of each component- laser tube and mirror holders was then adjusted with knobs to centre the alignment beam. Then, once the pump source was on, the second mirror was adjusted by rotating the mirror such that the reflected beam spanned a plane across the laser tube's Brewster window. Ignition of the beam occurred when the reflected alignment laser passed through the Brewster window with minute diffraction. This process was repeated between measurements to optimise the laser output.

Laser Stability

The stability of the He-Ne laser was tested for three different laser configurations. These were the combinations (with order left and right mirrors):

- HR flat/flat and HR flat/1000 mm
- HR flat/flat and HR flat/1400 mm
- HR flat/1000 mm and HR flat/1400 mm

HR stands for high reflectivity, and in this case all mirrors had a reflectivity $>90\%$. Note that the mirror specification gives left/right radii of curvature for each mirror. For each cavity the resonator length was changed between ranges of 500 mm-1300 mm at steps of 200 mm by moving the right mirror further down the optical rail, and once the beam was ignited instances of instability (flickering) were recorded. Note that in order to observe the phenomena for the HR flat/flat and HR flat/1400mm configuration, an extra measurement needed to be made at 1500mm. Results were hence compared to theoretical regions of stability.

Determining Wavelength

This analysis was conducted for both the He-Ne and the alignment laser. The resonator length was fixed at 500 mm at the left end of the optical rail. This time z_{gs} was varied, in the range 365mm-715mm at increments of 15mm. The length y_{s1}

was measured at each distance and was then plotted against z_{gs} in Excel. The slope of their trend line was found numerically. Subsequent mathematical analysis was then conducted to find the wavelengths and uncertainties, and thus compare to expected values.

Results & Discussion

Stable and Unstable points for each config are plotted below:

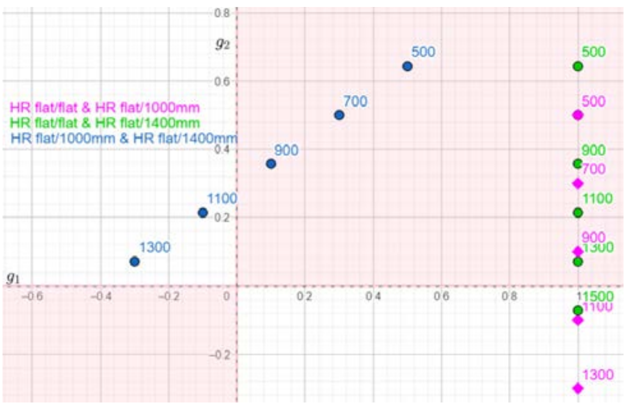


Figure 6: Plot of g_1 v/s g_2 for the 3 mirror configs. The stability region $0 < g_1 \cdot g_2 < 1$ is sketched in red and each data point is labelled by $d(mm)$ at which the measurement has been taken.

The figure above illustrates that the unstable points of the laser occur when resonator length is greater than the radius of curvature of the mirror (i.e. $d < b_i$). While this is unconditionally true for flat-concave configurations, it is only the partial truth for concave-concave cavities.
Extrapolation from the linear trend of the HR flat/1000 mm and HR flat/1400 mm case indicates that increasing the resonator length such that the distance is greater than the radius of curvature for both mirrors will map the data point to the region of stability in the third quadrant.
This result is supported by the theoretical understanding of resonator geometry. As is demonstrated in figure 3, instability occurs when the standing wave is disturbed. For the hemispherical case, the position of the flat mirror will steadily shift away from the focal length ($b/2$). Both mirrors are of the same size, so when $d = b$ the point of incidence will be in the same plane for either mirror, and increasing d after this point will mean that the beam escapes the cavity. For the concave-concave case the above also applies, but the curvature of the second mirror means that the shared focus point now also contributes to the stability [5]. Further experimentation beyond the scope of this experiment could be done to investigate the properties of this dual stability. While experimental values fall within the theoretical range, there was still some sources of error. In particular, the measurement was dependent on human observation and manual realignment. Inaccuracies in either of these due to human error are possible and would contribute to the observations of instability.

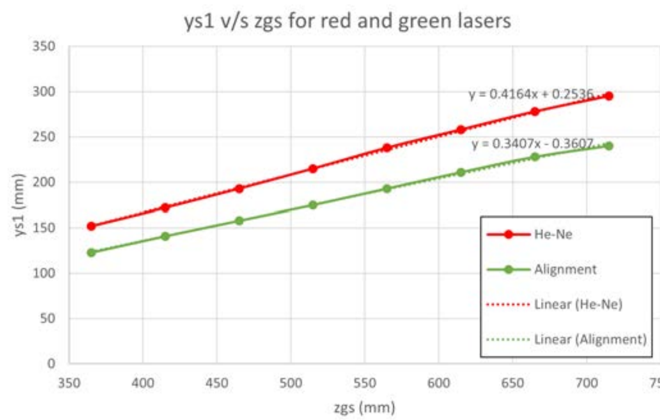


Figure 7: Plot of z_{gs} v/s y_{s1} for both the He-Ne Laser (Red) and the Alignment Laser (Green). Trend lines have also been plotted for each dataset as well as their equations.

For the He-NE Laser, Equation (3) gives

$$\frac{\Delta y_{s1}}{\Delta z_{gs}} = \tan(\alpha) = 0.4164 \rightarrow \alpha = 22.6°$$

Using the fact that $n = 1$, $l = 1.67\mu m$ from specifications of diffraction grating, Equation (2) gives

$$\lambda = l\sin(\alpha) = 642 \pm 7\, nm$$

Similarly, the Alignment laser has

$$\tan(\alpha) = 0.3407 \rightarrow \alpha = 18.8° \lambda = l\sin(\alpha) \quad = 539 \pm 8\, nm$$

where uncertainties are obtained by first using the LINEST function for the error of the slope and then using subsequent error propagation analysis.
Relevant literature places the wavelength of the He-Ne laser to be approximately 632.8 nm[4]. The addition of Brewster windows to the laser tube will also allow the transitions to occur at the higher wavelengths of $611.8nm$, $629.4nm$, $635.2nm$ and 640.1 nm[16]. In comparison, the transitions at 635.2 nm and $640.1nm$ are smaller than the experimental value, but fall within the range of uncertainty. Specifications for the alignment laser are also given to be 532 nm in the lab notes. The experimental value found is also a little high in comparison, but within the uncertainty range. The wavelength of emission is dependent on the spacing between lasing energy levels. A large wavelength is indicative of lower energy and hence a smaller spacing. As is seen in figure 7, the trend lines coincide with the obtained data to a high degree. The instances

of error can be attributed to human error and calibration error of the instruments used. Additionally, the spots ranged across some millimeters, and the centres of each were identified by observations which is prone to error. Misalignment and diffraction of the beam within the laser tube can also affect the diffracted spot.

Conclusion

The objective to experimentally obtain the stability threshold for different laser cavity configurations and determine the wavelength of lasers using the laser cavity geometry was fulfilled. The stability criterion was fulfilled as per theoretic estimations and was explained by the geometric properties of radii of curvature of mirrors and the resonator length. The resonator length needed to be smaller than the radii of curvature for stability, and configurations with two curved mirrors have two stable regions. Wavelengths of lasers (found to be $642nm$ for He-Ne and $539nm$ for alignment laser) were within theoretical ranges up to some uncertainty. Further investigations into the dual stability of the HR flat/$1000\,mm$ and HR flat/$1400\,mm$ cavity can be conducted in the future. Additionally, the influence of laser cavity geometry on other properties such as laser output power, beam waist and divergence can also be explored. Trend lines coincide to obtained data to a large degree. Sources of error include calibration error of instruments, misalignment of laser configurations and human error from observation.

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The End!

By EVERYONE

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